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Carbon emission trading and carbon taxes under uncertainties

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Abstract The idea of market-based carbon emission trading and carbon taxes is gaining in popularity as a global climate change policy instrument. However, these mechanisms might not necessarily have a positive outcome unless their value reflects socioeconomic and environmental impacts and regulations. Moreover, the fact that they have various inherent exogenous and endogenous uncertainties raises serious concerns about their ability to reduce emissions in a cost-effective way. This paper aims to introduce a simple stochastic model that allows the robustness of economic mechanisms for emission reduction under multiple natural and human-related uncertainties to be analyzed. Unlike standard equilibrium state analysis, the model shows that the explicit introduction of uncertainties regarding emissions, abatement costs, and equilibrium states makes it almost impossible for existing market-based trading and carbon taxes to be environmentally safe and cost-effective. Here we propose a computerized multi-agent trading model. This can be viewed as a prototype to simulate an emission trading market that is regulated in a decentralized way. We argue that a market of this type is better equipped to deal with long-term emission reductions, their direct regulation, irreversibility, and "lock-in" equilibria.

1 Introduction

The idea of carbon trading and taxation as a way of combating global climate change is gaining in popularity. At the same time, the uncertainties, both exogenous and endogenous (Rypdal and Winiwarter 2000; Winiwarter 2007; Lieberman et al. 2007; Marland 2008), inherent in carbon trading markets and taxes, raise serious concerns about their ability to contribute to controlling climate change in a cost-effective way.

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The international emission trading scheme under the Kyoto Protocol was devised to lower the cost of achieving greenhouse gas emission reductions for different countries: emissions are reduced where it is cheapest, and emission certificates are then traded to meet the nominal targets in each country (EEA 2006). Thus, in order to minimize costs and make environmentally safe decisions, parties can engage in a bilateral emission exchange process that is independent of market structures.

In contrast, carbon trading markets, which have become popular, resemble stock markets. Carbon markets, like other commodity markets, are volatile and are the result of and react to stochastic "disequilibrium" spot prices, which may be affected by speculations and bubbles (Energy Business Review 2006). The existing emission trading, therefore, might not necessarily minimize abatement costs and achieve emission reduction goals promoting environmental safety.

There are two main approaches to cost-effective pollution control: centralized "command-and-control" methods and decentralized market simulation schemes. If the centralized agency is fully informed about the emissions and abatement cost functions of all parties, finding emission levels that meet given environmental standards in a cost-effective way is a straightforward task. Unfortunately, parties prefer to keep the information private and the costs of emission reductions remain unknown. In the absence of information, a decentralized approach to cost-effective emission reduction is required, as in the bilateral emission trading scheme outlined in Section 2. An alternative to this scheme may be a tax scheme (Section 3) or a price-based scheme (Section 5) simulating a decentralized market solution.

The aim of this paper is to discuss a basic model for analyzing robust decentralized emission control schemes, which treats various uncertainties and detectability (Jonas and Nilsson 2007) of emissions explicitly. The model describes a trading system that enables parties to achieve solutions that are cost-effective and environmentally safe. Section 2 introduces the emission trading schemes and argues that the uncertainties and the way they are represented in emission trading significantly influence equilibrium prices. For example, the standard deterministic representation of emission uncertainties by equal-sided intervals might overlook their essential characteristics. These can, in turn, affect the timing when emission changes become detectable (i.e., when they outstrip the uncertainty associated with them) and whether or not emission reduction targets satisfy agreed safety controls, as illustrated in Appendix 1 and 2. In the sequential bilateral trading scheme of Section 3, the trade at each step takes place toward the cost-effective and environmentally safe equilibrium price. Section 3 also discusses the disadvantages of taxation. Section 4 outlines a computerized, multi-agent and decentralized trading system that allows irreversibility of emission trading to be coped with. Section 5 analyses path-dependencies of myopic trading schemes relying on instantaneous market prices. Section 6 concludes.

2 Trade equilibrium under uncertainty

Carbon control policies, like other environmental policies, should ideally be introduced so as to be environmentally safe and cost-effective. The following model (1)–(4) provides a basis for designing rather different decentralized emission trading schemes. Let us consider in detail an exchange scheme which does not require the existence of a market.

Our model reflects the following key features. The participants (countries, companies, or other emitting entities) are given a right to emit a specific amount for which they must hold an equivalent number of allowances or emission permits. The amount of emissions allowed is limited to the "cap" (Kyoto or other targets). If participants emit more than the cap, they are required to buy additional credits from the parties who pollute less. The transfer of permits is called "trading." Let us briefly discuss the deterministic model proposed in Godal et al. (2003) that will be further extended to include a stochastic model with probabilistic safety constraints. Uncertainties in emission trading have also been addressed in Nahorski et al. (2003) and Nahorski et al. (2007) where, similar to Godal et al. (2003), the uncertainty is added to the emissions reported in the compliance year before the target compliance is checked.

The decision problem of each party can be separated into two interdependent sub-problems. First, for a fixed amount of permits, each party solves its individual problem by deciding whether to spend resources on abating emissions or on investing in uncertainty reduction to satisfy emission targets. This problem does not require information from any other party. Second, each party needs to decide whether or not to exchange permits with other parties. This decision problem involves the cost functions of other parties. In the model this information is private and therefore the methodology of decentralized optimization (Ermoliev et al. 2000) is required.

2.1 Model with interval uncertainty

Let us consider first a model with interval uncertainties. For the individual optimization problem, we define the least costs $f_i(y_i)$ for party i (to comply with imposed targets with fixed permits y_i and the target K_i) as the minimum of emission reduction costs $c_i(x_i)$ and uncertainty reduction costs $d_i(u_i)$:

$$f_i(y_i) = \min_{u_i, x_i} \left[c_i(x_i) + d_i(u_i) \right], \tag{1}$$

$$x_i + u_i \leq K_i + y_i, \;\; x_i \geq 0, \;\; u_i \geq 0, \tag{2}$$

for all i, where x_i is the estimate of the reported emissions at source i, u_i is its uncertainty, and y_i is the amount of emission permits acquired by source i (y_i is negative if i is a net supplier of permits).

Remark 1 (*Long-term perspective*) In the model, we introduce a long-term perspective by explicit treatment of future uncertainties and a dynamic trading process. The environmental constraint (2) requires that estimated emissions plus their uncertainty undershoot the agreed emission target. This corresponds exactly to the detectability concept in Fig. 4.

The second optimization problem involves finding the permit vector $y = (y_1, ..., y_n)$ or distribution of permits minimizing the total or social costs

$$F(y) = \sum_{i=1}^{n} f_i(y_i) \tag{3}$$

subject to

$$\sum_{i=1}^{n} y_i = 0. \tag{4}$$

For illustration, let us assume that the cost functions $c_i(x_i)$ and $d_i(u_i)$ are positive, decreasing, convex in x_i and u_i, respectively, and continuously differentiable. Furthermore, if $f_i(y_i)$ is the minimum of two convex positive functions subject to a linear constraint, the function $f_i(y_i)$ is convex, positive, and decreasing. Then, from the Lagrangian minimization $\sum_{i=1}^{n} f_i(y_i) - \lambda \sum_{i=1}^{n} y_i$, a trade equilibrium can be defined as the vector $y = (y_1, ..., y_n)$ satisfying the following equations:

$$f_i'(y_i) = \lambda, \; i = \overline{1:n}. \tag{5}$$

The condition 5 states that, in equilibrium, the marginal value of a permit shall be equal to a specific unknown level (price) $\lambda = \lambda^*$ that is the same for all parties. At the equilibrium vector y^* the following condition holds true:

$$f_i(y) = \max_{x_i} \left[c_i(x_i) + d_i(K_i + y_i - x_i)\right] = \max_{u_i} \left[c_i(K_i + y_i - u_i) + d_i(u_i)\right].$$

Therefore from Eqs. 1, 2, it follows that at the equilibrium $y_i = y_i^*$, $\lambda = \lambda^*$, $x_i = x_i^*$, $u_i = u_i^*$:

$$c_i'(x_i) = d_i'(u_i) = \lambda, \tag{6}$$

where $\left(x_i^*, u_i^*\right)$ is the solution of the sub-problem (1), (2) for (y^*, λ^*), and $y^* = \left(y_1^*, ..., y_n^*\right)$ satisfying Eq. 5. This equation states that in the cost-effective and environmentally safe equilibrium, the marginal cost of holding emissions down to x_i^* will be equal to the marginal cost of holding uncertainty down to u_i^*. It is important to note that the explicit introduction of uncertainty u_i, the detectability of emissions, and safety constraints (2) into emission trading schemes may significantly affect the equilibrium and hence the design of emission trading schemes. In particular, it means that market prices must reflect Eqs. 5 and 6.

The price λ^* can also be viewed as a cost-effective and environmentally safe carbon tax. However, it is difficult for an agency acting as a central planer to know the cost functions of all parties and thus find equilibrium λ^*. In particular, if adopted by the agency, there is no guarantee that any fixed (regulated) price (tax) λ satisfies Eqs. 5 and 6. In contrast, the scheme of sequential bilateral trade (see Section 3) allows the equilibrium x_i^*, u_i^*, λ^* to be recovered without information on cost functions of all parties being known.

2.2 Probabilistic safety constraints

In the simplified case, constraints (2) assume that the uncertainty of emissions is characterized by equal-sided intervals. These constraints discount the level of reported emissions x_i by their uncertainty u_i (see Fig. 1). As the reduction of uncertainty (6) involves costs, the interval representation may incur costlier or even worst-case solutions, because it may not capture the likelihoods and the preference structure of individual emissions values within the given interval ranges.

In fact, Figs. 2 and 3 illustrate rather complex asymmetric variabilities of emissions and cases in which the interval representation of uncertainty might be inadequate. The following stochastic model introduces probabilistic safety constraints with risk-based discounting of reported emissions, which is less conservative than interval-based discounting.

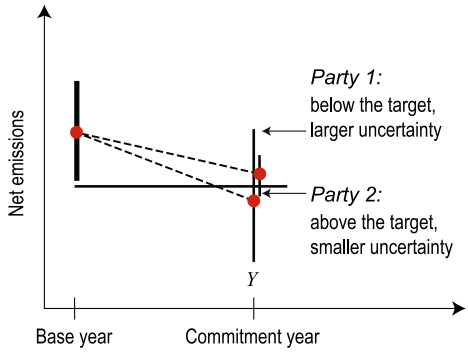

Fig. 1 Which party is more credible for emission trading? Party II reveals a smaller uncertainty interval, the mean of which, however, does not comply with the Kyoto target

Let us define the variability (uncertainty) of reported emission x_i as random variable $\xi_i(x_i, \omega_i)$. Then, the safety constraint can be written as a probabilistic version of the deterministic constraint (2):

$$P\left[x_i + \xi_i\left(x_i, \omega_i\right) \leq K_i + y_i\right] \geq Q_i \tag{7}$$

for all parties i, where Q_i is a safety level that ensures the probability of all potential emission paths to x_i, satisfying the emission target K_i exceeds Q_i.

A random variable $\xi_i(x_i, \omega_i)$ depends, in general, on x_i. In reality, the uncertainty ξ_i can be reduced by improvements to monitoring systems. Let us introduce for this purpose the variable u_i that may control the variability of emissions within the desirable safety level Q_i. If $z_i(x_i)$ is the minimal z such that

$$P\left[\xi_i\left(x_i, \omega_i\right) \leq z\right] \geq Q_i,$$

then the following equivalent constraints can be substituted for the safety constraint (7):

$$x_i + u_i \leq K_i + y_i, \;\; u_i \leq z_i\left(x_i\right). \tag{8}$$

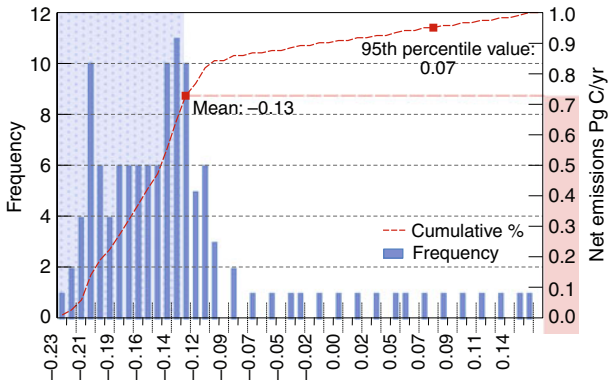

Fig. 2 Global CO_2 net terrestrial uptake, 1960–1970

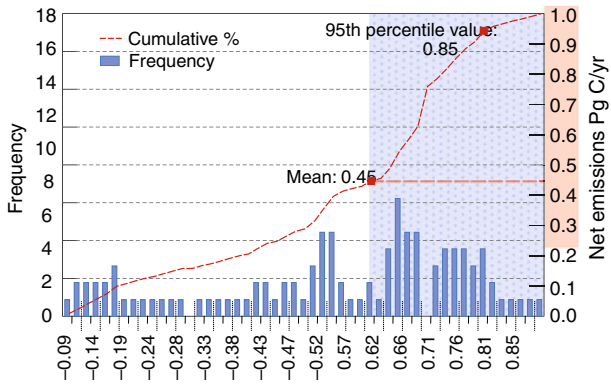

Fig. 3 Global CO_2 net terrestrial uptake, 1985–1995

For notational simplicity, the vector of all uncertainties affecting cost functions and emissions will be denoted by ω. We can redefine function $f_i(y_i)$ in Eq. 1 as

$$f_i(y_i) = \min_{x_i u_i} E\left[c_i(x_i, \omega) + d_i(u_i, \omega)\right], \tag{9}$$

where the minimization in Eq. 9 is subject to constraint (8).

Example 1 (*Deterministic safety constraints*) Assume that ξ_i does not depend on x_i. Then, the constraint (8) reduces to the constraint (2) of the original deterministic model which controls the safety of targets K_i only within a given safety level Q_i.

Example 2 (*Linear equivalent*) Often, $\xi_i(x_i, \omega_i)$ is represented as $\xi_i(x_i, \omega_i) = \gamma_i x_i + \varepsilon_i$, where $0 < \gamma_i < 1$, and ε_i is a random variable. The reduction of uncertainty is controlled by γ_i in the following manner. Let $\varepsilon_i(Q)$ be the minimal z such that $P[\varepsilon_i \le z] \ge Q_i$. Then the safety constraint (8) is reduced to linear constraints

$$x_i + u_i \le K_i - \varepsilon_i(Q_i) + y_i, \;\; u_i \le x_i.$$

After the individual sub-problems are solved, the optimal γ_i can be found as $\gamma_i = u_i/x_i$.

Remark 2 (*Risk indicator*) Safety constraints (7) are well known in financial applications as a Value-at-Risk indicator (Rockafellar and Uryasev 2000). Similar constraints are typically used in safety regulation by insurance companies, power plants, and in catastrophic risk management (Ermolieva and Ermoliev 2005).

The proposed models allow the comparative advantages of different emission control economic mechanisms to be analyzed.

3 Dynamic bilateral trading process and taxes

The overall goal of the parties participating in emission trading is to jointly achieve emission targets by redistributing the emission permits y_i, that is, to find a robust vector y that would minimize the risk-adjusted social costs of all parties (3) under safety constraints (8), where cost functions $f_i(y_i)$ are defined according to Eq. 9.

It is assumed that a party i knows its expected cost function $f_i(y_i)$, but that the expected social cost function $F(y)$ is unknown. As well as the uncertainty of the social cost function $F(y)$, this scheme takes into account uncertainties of cost functions $c_i(x_i)$, $d_i(u_i)$, which may be affected by market performance, production shocks, and technological uncertainties related to new technologies that cannot be known in advance.

The basic feature of the scheme (Ermoliev et al. 2000) is that two randomly selected parties exchange emission permits in a mutually beneficial way. A new pair is chosen and the procedure is repeated. The following simple equations illustrate that the bilateral exchange of emissions is beneficial for both parties. Let $y^k = \left(y_1^k, ..., y_n^k\right)$ be the vector of emission permits after k trades. Consider two parties i and j at step k with permits y_i^k and y_j^k.

According to Eq. 5, if any two parties i and j have different marginal costs on emission reduction $f_i'\left(y_i^k\right) \neq f_j'\left(y_j^k\right)$, then the permit vector $y^k = \left(y_1^k, ..., y_n^k\right)$ is not cost-efficient. Without loss of generality, assume that $f_i'\left(y_i^k\right) - f_j'\left(y_j^k\right) < 0$. Constraint (4) requires that the feasible exchange in permits has to be such that $y_i^{k+1} + y_j^{k+1} = y_i^k + y_j^k$. If we take $y_i^{k+1} = y_i^k + \Delta_k$ and $y_j^{k+1} = y_j^k - \Delta_k$, $\Delta_k > 0$, then the new feasible vector of permits y^{k+1} reduces the total costs of parties $f_i\left(y_i^k\right) + f_j\left(y_j^k\right)$ and hence the total cost $F(y^k)$:

$$\begin{aligned} F\left(y^{k+1}\right) - F\left(y^k\right) &= f_i\left(y_i^{k+1}\right) + f_j\left(y_j^{k+1}\right) - f_i\left(y_i^k\right) - f_j\left(y_j^k\right) \\ &= \Delta_k\left(f_i'\left(y_i^k\right) - f_j'\left(y_j^k\right)\right) + o\left(\Delta_k\right) < 0, \end{aligned} \tag{10}$$

for small Δ_k. This equation demonstrates that bilateral trade reduces the aggregate costs for sources i and j. We also have

$$f_i\left(y_i^{k+1}\right) - f_i\left(y_i^k\right) < f_j\left(y_j^k\right) - f_j\left(y_j^{k+1}\right). \tag{11}$$

That is, the new distribution of permits reduces costs of j more than increasing the cost of i. Hence j is able to compensate i for the increased costs in a mutually beneficial way.

A party j that decreases emission permit by $\Delta_k > 0$ may negotiate with party i such a level Δ_k that equalizes marginal costs, that is, $f_i'\left(y_i^k - \Delta_k\right) = f_j'\left(y_i^k + \Delta_k\right) = \lambda^k$, where λ^k is an equilibrium price (usually stochastic) at step k. Similar to Ermoliev et al. (2000) it can be proven that for convex functions $f_i(y_i)$, $f_j(y_j)$ the sequence of permits $y^k = \left(y_1^k, ..., y_n^k\right)$ and λ^k converges to an equilibrium satisfying Eq. 5. The computerized market system, described in the next section, allows more sophisticated global solutions for non-convex functions to be achieved.

It is important to compare the bilateral trading scheme outlined with price-based schemes and carbon taxes. A price or tax signal λ decentralizes the solution of overall minimization problem into individual sub-problems: find solutions $y_i(\lambda)$ minimizing functions $f_i\left(y_i\right) + \lambda y_i$. In general, solutions $y_i(\lambda)$ do not satisfy the balance Eq. 4, that is, as $\sum_{i=1}^n y_i\left(\lambda\right) \neq 0$, the price (tax) λ has to be adjusted toward the desirable balance. The common idea is to change current λ_k at time $k = 0,1,...$ proportionally to the imbalance, that is, $\lambda_{k+1} = \lambda_k + \rho_k \sum_{i=1}^n y_i\left(\lambda_k\right)$, with a proper step size ρ_k. It is unrealistic to assume that imbalances $\sum_{i=1}^n y_i\left(\lambda_k\right)$ can be evaluated exactly.

The inherent uncertainty of values $\sum_{i=1}^{n} y_i(\lambda_k)$ results in irregular adjustments of λ_k. It would thus be very difficult to establish an agency that is able to calculate imbalances and regularly update taxes to achieve harmonization across countries, thereby satisfying Eqs. 4–6. This would also include harmonization of taxes among all countries producing a similar product, which would be extremely difficult. Fundamental additional difficulties arise in the case of the market uncertainties analyzed in Section 5.

4 Computerized multi-agent decentralized trading system

Although during sequential bilateral trades, marginal costs and prices will differ for each sequential trade, the trading system will finally converge to an equilibrium where the marginal costs of all parties is equal to equilibrium price as in Eq. 5. This perfect trading system implies that trades being bilateral, sequential (dynamic), and random does not reduce the cost savings, even if sources only have information on their own abatement costs. However, there are major obstacles that can inhibit real markets from perfect functioning according to the proposed procedure. In a perfect market, a party that has sold permits at an early stage of the trading process would be able to renegotiate its earlier transaction. In the real emission trading market, this type of counteraction may be impossible because decisions are irreversible: investments may already have been made, and these investment costs are largely sunk costs. This is the fundamental obstacle involved in the design of cost-minimizing and environmentally safe emission trading markets. Available computer technology and numerically stable optimization procedures allow a computerized (say, Web-based) Multi-Agent Decentralized Trading System to be organized to resolve these issues.

One can imagine a distributed computer network that connects computers of parties with the computer of a central agency. The party anonymously stores information on its specific cost functions and other characteristics of the underlying optimization model 8, 9, including specific probability distributions. The central agency stores information on the emission detection model. The computer of the central agency generates a pair of parties i, j and anonymously "negotiates" with the computers of these partners a proper Δ_k that solves the sub-problem (12). This can easily be done without revealing parties' private information, and the process is repeated until equilibrium levels have been reached. This procedure allows an equilibrium solution to be found that can then be implemented in reality. It is important to stress that a network of interconnected computers is essential for the rapid, smooth, and robust functioning of the emission trading market. There would also be a clear separation between a first stage, in which provisional bids are made between the computers of the parties and a reconstruction of the decisions is still allowed, and a second stage, when contracts have been concluded and investments in emission control have been implemented.

It is well known (Baumol and Oates 1971) that the market will not usually generate desirable outcomes if market prices fail to reflect socioeconomic and environmental impacts. In such a case, it is typically necessary to establish negotiation processes between parties involved to determine desirable collective solutions. From this perspective, the computerized Multi-Agent Decentralized Trading system can

be viewed as a device for collective negotiations and decision making in the presence of inherent uncertainties and irreversibilities.

5 Myopic market processes

The basic model 3, 4, 8, 9 takes a long-term perspective on emission permit trading. Parties use rational expectations and safety constraints to achieve cost-effective and environmentally efficient outcomes that are robust against potential future eventualities. The resulting trading scheme is similar to a pure exchange economy. There are no demand and supply functions. Instead, the safety constraints oblige parties to invest in emission and uncertainty reductions and consequently act as suppliers of emission permits until a rational expectations equilibrium emerges, that is, sequential decisions on the part of parties generate a path of emission permit prices λ_k that converge to the equilibrium price.

The proposed model exists in a prototype "world" where in a perfect market the parties would be able to trade at equilibrium price. In the actual emission trading market, the emissions trades are accomplished at spot disequilibrium prices, and once done, cannot be redressed, which may involve potential lock-in irreversibility not only of trades but also of technological investments. The short- term price-driven market perspective orient parties to instantaneous information ω on prices and cost functions. At time interval k, parties observe uncertainty ω_k and thus know their own cost functions $c_i(x_i, \omega_k)$, $d_i(x_i, \omega_k)$. Based on this information, they trade permits until the next time interval $k + 1$ when new information comes on stream. In other words, parties calculate cost functions

$$f_i\left(y_i, \omega_k\right) = \min_{u_i, x_i}\left[c_i\left(x_i, \omega_k\right) + d_i\left(u_i, \omega_k\right)\right]$$

subject to the safety constraints 8, and they minimize $\sum_{i=1}^{n} f_i\left(y_i, \omega_k\right)$ subject to $\sum_{i=1}^{n} y_i = 0$.

This yields ω_k-dependent decisions $x_i(k, \omega_k)$, $u_i(k, \omega_k)$, $y_i(k, \omega_k)$. At time interval $k + 1$, a new observation $\omega_k + 1$ may contradict ω_k, requiring significant revisions to be made to these decisions, which may then be impossible because of their irreversibility. The sequence of myopic short-term decisions $x_i(k, \omega_k)$, $u_i(k, \omega_k)$, $y_i(k, \omega_k)$ would exhibit path-dependent random behavior without providing an equilibrium.

6 Concluding remarks

The feasibility of carbon emission trading and carbon taxes is usually discussed under strong assumptions that all actions are made simultaneously at known equilibrium prices, which implies the existence of a perfectly informed central agency. The proposed sequential bilateral trading scheme under uncertainties avoids this assumption. Sequential trades and resulting emission prices implicitly depend on cost functions and safety constraints for environmental targets. With probabilistic safety constraints, the parties set the level of their exposure to uncertainties and risks. The safety constraints discount the reported emissions to verifiable levels. Yet there is a serious concern that the irreversibility of trades prevents these schemes from achieving cost-effective and environmentally safe solutions. This calls for the

use of a computerized multi-agent emission trading system that provides a collective decentralized regulation of trades.

The computational effectiveness and counterintuitive effects of deterministic interval uncertainties on emission trading are analyzed in Godal et al. (2003) using data on the European Union countries, Russia, Ukraine, and the USA. The modification of these large-scale calculations for stochastic models is a straightforward but tedious task that is beyond the scope of this paper. In general, the stochastic model allows less conservative confidence intervals to be substituted for deterministic uncertainty intervals, leading to less conservative risk-adjusted conclusions.

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Appendix 1: Uncertainties and trends of carbon fluxes

Here, we illustrate the role of emissions uncertainties and the need for their proper representation in the context of the Kyoto Protocol. The main question that Kyoto parties will face at the end of the commitment period is whether they have fulfilled their obligations. As can be seen from Fig. 1 (see also Jonas and Nilsson 2007), situations might arise when it is difficult to determine which Kyoto parties have genuinely met their Kyoto targets and which Kyoto parties are more "credible," especially when it comes to emission trading.

Uncertainties in Fig. 1 are represented by means of symmetrical intervals. In reality, however, emissions might have different likelihoods within these intervals (i.e., rather general skewed probability distributions). In this case, the use of equal-sided uncertainty representation means that essential patterns of emission changes might not be considered, as illustrated in Figs. 2 and 3. These figures show the variability in emissions of the global CO_2 net terrestrial uptake (see also Ermolieva et al. 2007, data source: http://lgmacweb.env.uea.ac.uk/lequere/co2/carbon_budget.htm). The histogram in Fig. 2 is skewed to the left (more frequent values are on the left-hand side). In the next study period, Fig. 3, the situation changes: more values are concentrated on the right-hand side. Between the two periods, the system changed from being a source to being a sink of CO_2. Moreover, while in Fig. 2, the rare values on the right-hand of the mean have probability only in the order of 15–20%, in Fig. 3, the values and their likelihoods on the right-hand of the mean have much heavier probability mass and, therefore, would be essentially more important for deciding about the level of emission permits to trade. It is clearly impossible to represent the uncertainty characteristics of the variable in Figs. 2 and 3 through intervals.

Appendix 2: Detectability of emissions

Apart from detecting natural variability and changes of emissions as illustrated in Figs. 2 and 3, the detectability of emission changes may also be applied to the analysis of human-related uncertainties, e.g., associated with underreported emissions. The simplest way to introduce the detectability concept into the emission trading schemes (models) is to make a straightforward representation of uncertainties by equal-sided intervals setting ranges of potential emissions as in Section 2.1 (for overview, see

Jonas et al. 1999). Such a representation allows only a conservative or, in a sense, "worst-case" conclusions to be derived, as it does not really give a clue as to the different likelihoods of emissions within the interval (i.e., rather different left- or right-skewed probability distributions; for example, as in Figs. 2 and 3, may be "covered" by the same interval). Let us illustrate the main idea of the interval-based detection techniques with a simple example in Fig. 4. Assume that uncertainty of emission e_1 in the base year t_1 is characterized by equal-sided interval $[e_1 - \varepsilon, e_1 + \varepsilon]$. The uncertainty of reported emission e_2 in the commitment year t_2 ($t_1 < t_2$) is characterized by the same type of interval $[e_2 - \varepsilon, e_2 + \varepsilon]$. We assume that $e_1 > e_2$, although the case $e_1 < e_2$ is also possible (e.g., as a result of emission trading). The emission changes are said to be detectable at time t_2, if the change in carbon emissions $\Delta e = e_1 - e_2$ at time t_2, with reference to time t_1, is greater than the uncertainty (e.g., $e_2^{\max} - e_2$, of the reported net carbon emissions at time t_2). Section 2.2 deals with this case within a stochastic emission trading model, where $e_1 \leq K_i + y_i$, $e_2 \leq x_i$, $\varepsilon = u_i$ in Eqs. 1 and 2.

Under the non-restrictive assumption that first-order linear approximations for emission $e(t)$ and uncertainty $\varepsilon(t)$ trends are applicable for $t_1 \leq t \leq t_2$, the detection time t^* is then defined as the first time moment at which net emission change Δe outstrips the uncertainty interval ε.

The deterministic detection concept allows describing the dynamics of emissions and their uncertainties, depending on data, up to any order. In Fig. 4 emissions are described up to the first order and uncertainties by a constant in absolute (or relative) terms. That is, the figure's focus is on two points in time, the base year and the commitment year. However, this concept always assumes a single scenario (here a straight line (e_1, e_2)) of how emissions and their uncertainties evolve from a value within an uncertainty interval at t_1 to a value within an uncertainty interval at t_2. As Figs. 2 and 3 show, emissions within uncertainty intervals may have rather different skewed probability distributions.

Remark 3 (*Stochastic detection models*) The detection of emission changes accounting for non-symmetrically distributed emissions is addressed by the stochastic detection technique (DT) proposed in Hudz (2002) and Hudz et al. (2003), and Ermolieva et al. (2007). Emission uncertainties and stochastic detection techniques are also discussed in Nahorski et al. (2003). The goal of the stochastic DT is to rank the trading parties by a safety indicator, which represents the percentile (probability) of emission

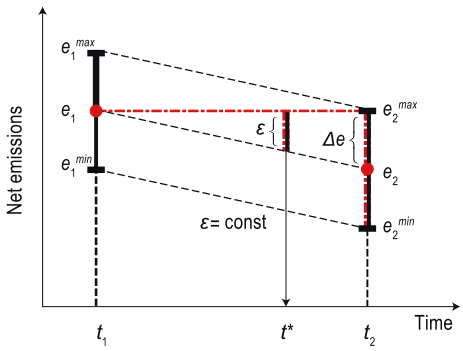

Fig. 4 Simplified illustration of detection time t^*

changes detectable in a given time period. Stochastic emission trading model in Section 2.2 allows the deterministic uncertainty intervals to be replaced by possibly much smaller, confidence (safety) intervals. Comparative analysis of deterministic and stochastic detection techniques using simplified version of the detection model can be found at http://www.iiasa.ac.at/Research/FOR/unc_prep.html and educational software at http://www.iiasa.ac.at/Research/FOR/vt_concept.html).

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